A Mathematical Model of the Dimensional Chain for a Generation 2 Wheel Hub Unit

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Abstract *This article overviews wheel hub design solutions and proposes a mathematical model of the dimensional chain and a tolerance formula for calculating axial clearance for a generation 2 wheel hub assembly with ball bearings. The dimensional chain analysis and its synthesis are carried out using three partial interchangeability methods. The possibility of manufacturing the hub bearing using selection compensation was proposed. The considerations made provide an alternative to the current method of process design based on numerous trials and considerable cost.*

Keywords *rolling-element bearings, dimensional chain, tolerance formula, axial clearance, wheel hub unit*

Highlights
- *A mathematical model of the dimensional chain and the tolerance equations for axial clearance was developed.*
- *Axial clearance and its limit deviations were compared to the constructor's values, revealing significant discrepancies.*
- *Calculations for component dimension tolerances show that achieving partial interchangeability in production is not feasible.*
- *Calculated tolerances of independent dimensions using selection compensation into 9 selection groups.*

1 INTRODUCTION

The main objective of this paper is to present an approach to the design of manufacturing processes using a mathematical model of the dimensional chain to achieve the expected axial clearance. In previous practice when designing bearing hubs, designers did not build mathematical models of dimensional chains. Tolerances were selected intuitively based on the knowledge and experience of the designer. They were often suboptimal and economically unjustified. Therefore, work was undertaken to build a mathematical model of the dimensional chain of the axial clearance of a second-generation ball bearing hub assembly, in order to use it to verify the hub design and to plan the optimal manufacturing process. The hypothesis was made that in cases where the tolerance values obtained using the model would be less than the tolerances adopted by the designer, it would be necessary to change the tolerances of the independent dimensions and manufacture the product under interchangeable conditions using a selective compensation approach.

Tolerance design is very important in product development and manufacturing processes. It is particularly important when applying the concept of concurrent engineering (CE). This ensures that manufacturing costs are minimised with maximum product quality **[1]**. Many articles address the issues of tolerancing the dimensions of independent parts as well as assembling them into assemblies with the indication of problems in building tolerance chain models **[2]**. Singh et al **[3]** and **[4]** characterise various studies and theories on tolerance analysis and synthesis, which can be divided into traditional and advanced ones. Traditional tolerance design approaches do not consider the impact of tolerance on manufacturing cost as opposed to advanced ones. The authors of the theoretical considerations have not supported their application in industry with examples. Tolerance optimisation for products with multidimensional chains was described by Tsung **[5]** using the example of a ball bearing incorporated into a bicycle bottom bracket using specialised software. Nonetheless, the measurement of components at the manufacturing stage and their association at the assembly stage do not reflect the problems of determining the independent dimensional tolerances themselves.

There are many methods of tolerance analysis in use today, both manual and software-based. The use of computer software is constantly being developed and the results obtained can be used for analysis and optimal design **[6]** and **[7]**. In this paper, however, manual methods have been used to determine the dimensional chain model, perform its analysis and synthesise it for the axial clearance of a generation II hub.

Fig. 1. Wheel hub unit with two tapered roller bearings

Bearing hubs have undergone extensive modifications over the years in order to reduce their manufacturing costs and meet market expectations. Decisions made at the design stage help minimize the

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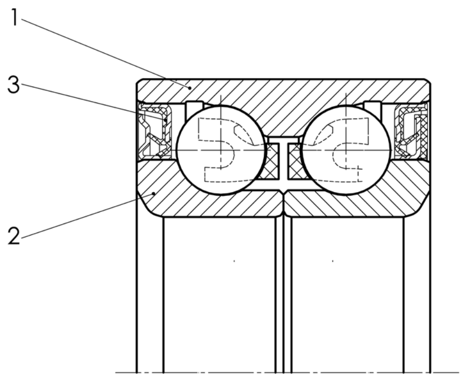

Fig. 2. Generation I wheel hub unit with a double-row angular contact ball bearing; 1 outer ring, 2 inner ring, and 3 seal

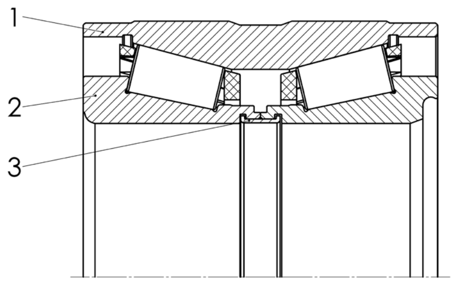

Fig. 3. Generation I wheel hub unit with a double-row tapered roller bearing; 1 outer ring, 2 inner ring and 3 clamping ring

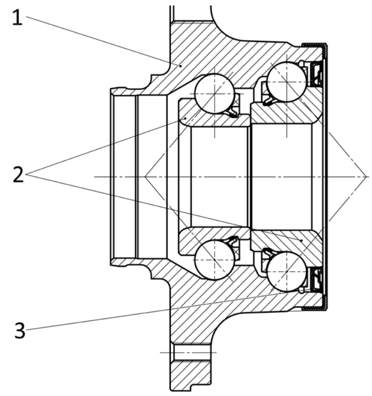

Fig. 4. Generation II wheel hub unit based on the ball bearing design; 1 outer flanged ring, 2 inner rings, and 3 seal

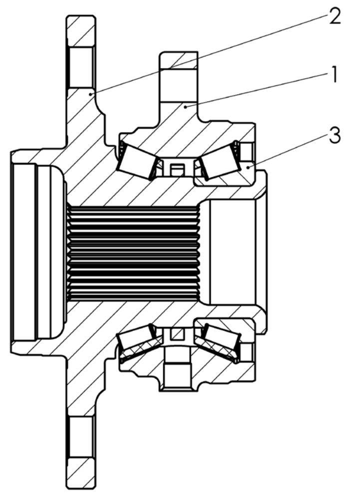

Fig. 5. Generation III wheel hub unit based on the tapered roller bearing design; 1 outer flanged ring, 2 inner flanged ring and 3 standard inner ring

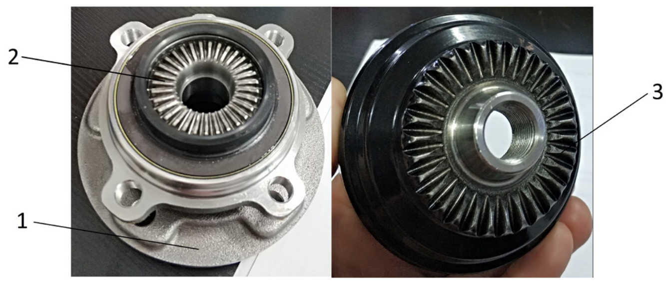

Fig. 6. Generation III wheel hub unit with a torque transmission gear system; 1 outer flanged ring, 2 inner flanged ring with a face gear and 3 halfshaft gear with meshing teeth

risks of product failure, which may result from errors associated with drawing interpretation, product manufacturing, or its primary or secondary assembly. The major factors responsible for these errors are the designer's insufficient knowledge of the processes involved in the manufacturing of the product and a low level of mechanization or automation of the product assembly [8]. Designers making decisions on tolerance sizes generally rely on intuition and experience. Such an approach, however, may not be correct because if tolerances are too small, the manufacturing costs increase substantially.

Wheel hub units for motor vehicles are precision products where appropriate dimensioning and tolerancing of components are critical aspects of engineering design. The simplest wheel hub unit configuration, depicted in Fig. 1, features a pair of tapered roller bearings in an adjusted face-to-face preload arrangement.

Another design solution is the first-generation wheel hub unit, fitted with a double-row angular contact ball bearing (Fig. 2) or a double-row tapered roller bearing (Fig. 3), lubricated for life with defined and preset clearance.

The second generation unitized or integrated wheel hub unit illustrated in Fig. 4 is a solution in which the bearing housing is replaced by an outer ring with a flange to which the rotating parts of the brake system and the road wheels are attached. It comes in two variants based either on the ball bearing or tapered roller bearing designs [9].

A more technologically advanced solution is the third-generation wheel hub unit with ball or tapered roller bearings (Fig. 5), where the outer flanged ring (1) takes over the function of the housing, and the inner flanged ring (2) allows the attachment of the brake disc and the road wheel. The wheel hub unit design has undergone various modifications to suit specific purposes. For example, the unit shown in Fig. 6 features a torque transmission gear system with an inner flanged ring (2) and a halfshaft gear (3).

When designing wheel hub units, engineers need to determine the tolerance limit for each toleranced dimension. This requires robust design knowledge and previous experience. They also analyze all the relevant measurement and simulation data. In some cases, however, tolerances assumed by the designer may not ensure that parts will function correctly [10]. Measurement methods used at the manufacturing stage play an essential role [11] and [12]. These days, geometrical tolerances can also be used to precisely define the product, such as for the design of gears [13]. This methodology, called geometrical product specification (GPS), described by EN ISO 8015:2012 [14] and ISO 492:2014 [15] has supplemented the traditional techniques for determining the geometrical requirements of engineering products. GPS helps specify linear and angular dimensional tolerances, geometrical tolerances, and surface texture tolerances [16] and [17].

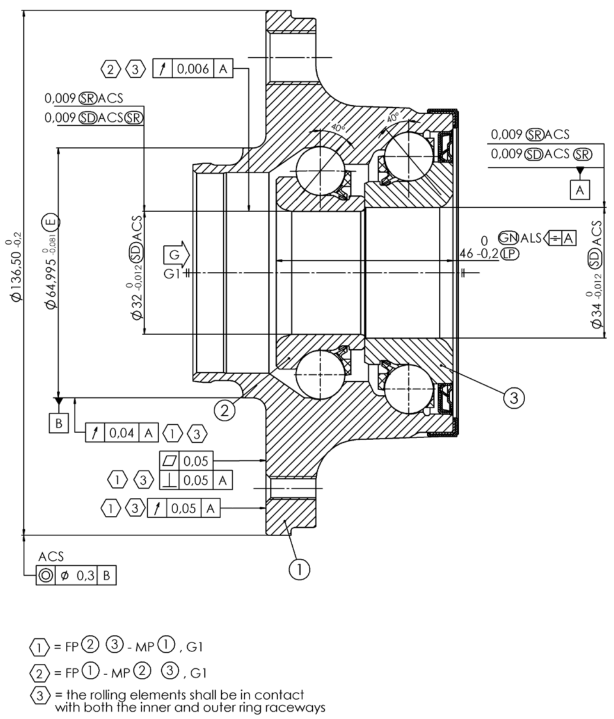

Fig. 7. **Generation II wheel hub unit with a double-row angular contact ball bearing**

In the study, the GPS dimensioning principles were employed to develop the design of a second generation wheel hub unit incorporating a double-row angular contact bearing (Fig. 7) [18].

2 A MATHEMATICAL MODEL

A mathematical model of the tolerance formula for the axial clearance of the generation 2 wheel hub unite with a double-row angular contact ball bearing is presented. The components of a double-row angular contact bearing, such as the inner rings, the outer ring, and the balls need to be carefully selected to ensure correct axial and radial clearance. In practice, however, for this type of bearing, only the axial clearance is measured under axial load, which is exerted in the axial direction on the outer ring alternately on either face, with the inner rings held stationary.

Assuming zero axial load, we obtained a dimensional chain with 14 components:

- m is axial distance between the raceways of the outer ring,
- n_l axial distance between the raceway of the left inner ring and its narrow face,
- n_p axial distance between the raceway of the right inner ring and its narrow face,
- r_{wl} radius of the left inner ring raceway,
- r_{wp} radius of the right inner ring raceway,
- r_{zl} radius of the left outer ring raceway,
- r_{zp} radius of the right outer ring raceway,
- d_{wl} diameter of the left inner ring raceway,
- d_{wp} diameter of the right inner ring raceway,
- d_{zl} diameter of the left outer ring raceway,
- d_{zp} diameter of the right outer ring raceway,
- α contact angle,
- d_l diameter of the left raceway ball,
- d_p diameter of the right raceway ball.

Thus, the dimensional chain equation for axial clearance is a function [19]:

$L = f(m, p_l, p_p, r_{wl}, r_{wp}, r_{zl}, r_{zp}, d_{wl}, d_{wp}, d_{zl}, d_{zp}, \alpha, d_l, d_p)$.

Because of the symmetry, the number of dimensions was reduced to half. Thus, the dimensional chain was simplified to an eight-component chain. The function had the form:

$L = f(m, n, r_w, r_z, d_w, d_z, \alpha, d)$,

where m is axial distance between the raceways of the outer ring, n axial distance between the raceways of the inner rings, r_w radius of the inner ring raceway, r_z radius of the outer ring raceway, d_w diameter of the inner ring raceway, d_z diameter of the outer ring raceway, α contact angle, and d ball diameter. In the study, a simplified design of the wheel hub unit (Fig. 8) was considered.

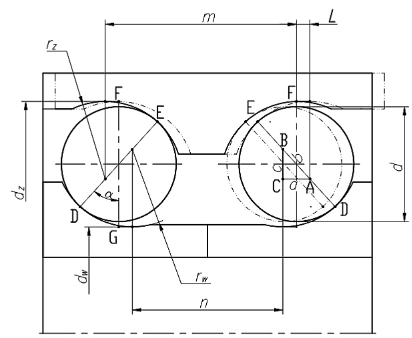

Fig. 8. **Simplified design of the wheel hub unit with a double-row angular contact ball bearing**

The dimensional chain equation for the axial clearance was written as:

$$m = n + 2a - L. \tag{1}$$

Hence,

$$L = n - m + 2a, \tag{2}$$

$$a = \sqrt{b^2 - c^2}, \tag{3}$$

$$b = r_w + r_z - d, \tag{4}$$

$$c = r_w + r_z - 0.5(dz - dw), \tag{5}$$

$$a^2 = (r_w + r_z - d)^2 - (r_w + r_z - 0.5(d_z - d_w))^2. \tag{6}$$

Let

$$R = r_w + r_z \quad \text{and} \quad S = 0.5(d_z - d_w). \tag{7}$$

After substituting Eq. (7) into Eq. (6), we obtaine

$$a^2 = (R - d)^2 - (R - S)^2, \tag{8}$$

and after the conversion, we had

$$a^2 = (d - S)(d + S - 2R). \tag{9}$$

The equation of the dimensional chain for the axial clearance was thus given as

$$L = n - m + 2\sqrt{(d - S)(d + S - 2R)}\,. \tag{10}$$

The tolerance formula was derived using the partial interchangeability method. It was assumed that the coefficient of variation for all the components of the dimensional chain was equal to 1. The tolerance formula for this complex chain was determined from Eq. (11):

$$T_L^2 = \sum_{i=1}^{k-1} \left(\frac{\partial L}{\partial A_i} \right)^2 TA_i^2. \tag{11}$$

The partial derivatives were calculated as:

$$\frac{\partial L}{\partial n} = 1, \quad \frac{\partial L}{\partial m} = -1, \tag{12}$$

$$\frac{\partial L}{\partial d} = \frac{2(d - R)}{\sqrt{(d - S)(d + S - 2R)}}, \tag{13}$$

$$\frac{\partial L}{\partial S} = \frac{2(R - S)}{\sqrt{(d - S)(d + S - 2R)}}, \tag{14}$$

$$\frac{\partial L}{\partial R}=\frac{2(S-d)}{\sqrt{(d-S)(d+S-2R)}}. \quad (15)$$

Since

$$\beta=\sqrt{(d-S)(d+S-2R)}, \quad (16)$$

the tolerance equation took the following form:

$$T_L^2=T_n^2+T_m^2+\frac{4}{\beta^2}\left[(d-R)^2T_d^2+(R-S)^2T_S^2+(S-d)^2T_R^2\right], \quad (17)$$

$$T_L=\sqrt{T_n^2+T_m^2+\frac{4}{\beta^2}\left[(d-R)^2T_d^2+(R-S)^2T_S^2+(S-d)^2T_R^2\right]}. \quad (18)$$

3 VALIDATION OF THE MATHEMATICAL MODEL

In this paper, the validation of the mathematical model of the axial clearance dimension chain is reduced to analysis and synthesis to confirm that the equation is mathematically and physically correct and meets the specified functional requirements.

The nominal value of the axial clearance in the analysed wheel hub unit was calculated using Eq. (18), derived under the assumption of partial interchangeability. The assumption was made because there were eight components in the dimensional chain and the designer's past experience in the design and production of similar products indicated such an approach was correct. Table 1 summarizes the nominal values of the dimensions shown in Fig. 8 together with their tolerances.

Table 1. Nominal values and tolerances determined for the dimensions provided in Fig. 8

Dimension	Unit	Nominal value	Tolerance
$n=n_l+n_p$	[mm]	22.772	± 0.020
m	[mm]	23.488	± 0.010
r_z	[mm]	6.730	± 0.025
r_w	[mm]	6.540	± 0.025
d	[mm]	12.700	± 0.00975
α	[°]	40	-
d_z	[mm]	65.878	± 0.02
d_w	[mm]	40.211	± 0.02
L	[mm]	0.015	± 0.010

Due to the complex nature of the eight-component dimensional chain, small tolerances, and measurement difficulties, the tolerance of the contact angle α was assumed to be zero.

3.1 Dimensional Chain Analyis

Eq. (7) was used to calculate the values of R and S with tolerances.

$R=6.54^{\pm0.025}+6.73^{\pm0.025}=13.27^{\pm0.05}$ mm,

$S=0.5(65.878^{\pm0.02}-40.211^{\pm0.02})=12.834^{\pm0.02}$ mm.

The numerical values from Table 1 and the calculated values of R and S from Eq. (7) were substituted into Eq. (10) to get the nominal value of the axial clearance, $L = 0.0183$ mm

$$L=22.772-23.488+2\times\sqrt{(12.7-12.834)(12.7-12.834-26.54)}=0.0133\text{ mm}.$$

The method of differential calculus, Eqs. (19) and (20), was employed to calculate the limit deviations.

$$x_2=\sum_{i=1}^{k}\frac{\partial L}{\partial A_i}a_{2i}+\sum_{i=1}^{n}\frac{\partial L}{\partial B_i}b_{1i}, \quad (19)$$

$$x_1=\sum_{i=1}^{k}\frac{\partial L}{\partial A_i}a_{1i}+\sum_{i=1}^{n}\frac{\partial L}{\partial B_i}b_{2i}. \quad (20)$$

Then, the values of the partial derivatives were determined using Eq. (12 to 15) and Eq. (16) in order to calculate β.

$$\beta=\sqrt{(12.7-12.834)(12.7+12.834-26.64)}=0.3672.$$

Thus

$$\frac{\partial L}{\partial d}=\frac{2(12.7-13.27)}{0.3672}=-3.105,\quad \frac{\partial L}{\partial S}=\frac{2(13.27-12.837)}{0.3672}=2.375,$$

$$\frac{\partial L}{\partial R}=\frac{2(12.834-12.7)}{0.3672}=0.73.$$

The upper limit deviation x_2 was:

$x_2=0.02+2.375\times0.02+0.73\times0.05+0.01+(-3.105)\times(-0.00975)$
$=0.144$ mm,

while the lower limit deviation x_1 was:

$x_1=-0.02+2.375\times(-0.02)+0.73\times(-0.05)-0.01$
$+(-3.105)\times0.00975=-0.144$ mm.

The axial clearance determined from the tolerances of the components of the dimensional chain assumed by the designer was:

$L=0.0183^{\pm0.144}$ mm

(with $L=0.015^{\pm0.010}$ mm assumed by the designer).

As can be seen, the calculated axial play was 3.3 µm greater than that assumed by the designer. The calculated tolerance value did not correspond to the nominal value of the axial clearance.

3.2 Synthesis of the Dimension Chain

The tolerances of the component dimensions were calculated for partial interchangeability using the method of equal tolerance, equal tolerance class and equal influence [20]. The experience of bearing manufacturers shows that performing these calculations for total interchangeability gives tolerance results of independent dimensions so small that it is not possible to meet them during technological process execution.

3.2.1 Method of Equal Tolerance

By substituting the numerical data into the tolerance equation; Eq. (18) and assuming $T_n=T_m=T_d=T_S=T_R=T$ the result was obtained:

$$T_L=\sqrt{T_n^2+T_m^2+\frac{4}{\beta^2}\left[(d-R)^2T_d^2+(R-S)^2T_S^2+(S-d)^2T_R^2\right]}$$

$$=\sqrt{T^2+T^2+\frac{4}{\beta^2}\left[(d-R)^2T^2+(R-S)^2T^2+(S-d)^2T^2\right]}$$

$$=T\sqrt{2+\frac{4}{\beta^2}\left[(d-R)^2+(R-S)^2+(S-d)^2\right]}$$

$$=T\sqrt{2+\frac{4}{0.135}[0.325+0.19+0.018]},$$

$$T=\frac{0.02}{4.218}=0.005\text{ mm}.$$

3.2.2 Method of Equal Tolerance Class for a Complex Chain

Eqs. (21) and (22) were used to calculate tolerances using the equal tolerance class method.

$$k_L^2T_L^2=a^2\sum_{i=1}^{n-1}k_{A_i}^2\left(\frac{\partial L}{\partial A_i}\right)^2\left(\sqrt[3]{A_i}\right)^2, \quad (21)$$

$$T_{A_i}=a\sqrt[3]{A_i}. \quad (22)$$

By substituting the values from Table 1 into Eq. (22), the results were:

$T_n=2.834a,\ T_m=2.862a,\ T_d=2.333a,\ T_S=2.341a,\ T_R=2.368a.$

After substitution into Eq. (21), the following was obtained

$$T_L{}^2 = a^2(1\times2.834^2+1\times2.862^2+9.641\times2.333^2+5.641\times2.341^2 + 0.533\times2.368^2) = a^2\times102.6, \text{ hence,}$$

$$a = \frac{T_L}{\sqrt{102.6}} = 0.00197 \text{ mm.}$$

So the tolerances of the component dimensions are respectively: T_n=0.006 mm, T_m=0.006 mm, T_d=0.005 mm, T_S=0.005 mm, and T_R=0.005 mm.

3.2.3 Method of Equal Impact

Based on Eq. (23) for the composite chain and assuming that the components of the chain have a normal distribution i.e. k = 1, the result was obtained:

$$k_{A_1}^2\left(\frac{\partial L}{\partial A_1}\right)^2 T_{A_1}^2 = k_{A_2}^2\left(\frac{\partial L}{\partial A_2}\right)^2 T_{A_2}^2 = \ldots = k_{A_n}^2\left(\frac{\partial L}{\partial A_n}\right)^2 T_{A_n}^2 = m^2. \quad (23)$$

hence $T_{A_n} = \sqrt{\dfrac{m^2}{\left(\dfrac{\partial L}{\partial A_n}\right)^2}} = \dfrac{m}{\left|\dfrac{\partial L}{\partial A_n}\right|}$.

For five dimensions of the chain $T_L{}^2 = 5\times m^2$,

hence $m = \dfrac{T_L}{\sqrt{5}} = \dfrac{0.02}{2.236} = 0.009$ mm.

The tolerances of the individual dimensions are therefore T_n=0.009 mm, T_m=0.009 mm, T_d=0.003 mm, T_S=0.004 mm, and T_R=0.012 mm.

Table 2. Summary of results for three methods of calculating tolerances for partial interchangeability

Tol. specified by constructor		Tol. calculated using the equal tolerance method	Tol. calculated using the equal tolerance class method	Tol. calculated using the equal impact method
T_n [mm]	0.04	0.005	0.006	0.009
T_m [mm]	0.02		0.006	0.009
T_d [mm]	0.0195		0.005	0.003
T_S [mm]	0.04		0.005	0.004
T_R [mm]	0.10		0.005	0.012

Table 2 summarises the results for the three methods mentioned above and the values adopted by the designer. No calculations have been made for the minimum cost method because the cost functions of making the rings as a function of changes in the dimension chain values are not known.

3.3 Partial Interchangeability for Axial Clearance with Selective Compensation

For small tolerances of the dependent dimension, it often turns out that the tolerances of the independent dimensions determined under the assumption of partial interchangeability are very small. Meeting them results in a significant increase in manufacturing costs or is even impossible. In turn, their enlargement generates an unacceptable number of defects. In such cases, the most economical solution is to make the individual parts within the tolerances achievable under the given production conditions. The tolerance of the dependent dimension will then be bigger than the tolerance assumed by the designer, and additional work must be done during assembly to achieve the resulting dimension within the assumed limits. This kind of interchangeability, in which additional operations are needed to obtain the correct product, is called conditional interchangeability. One of its types is the selection of parts with the right dimensions - selection compensation. It involves measuring all the parts to be made and dividing them into several dimensional groups and associating them appropriately together with the rolling elements during assembly [21] In the case of selection interchangeability, dividing all the parts from a given lot into m selection groups allows their production tolerance to be increased m times. We can therefore write that the actual closure dimension tolerance $T_L{}'$ is equal to the product of the number of selection groups m and the closure dimension tolerance in a given selection group T_L [22].

$$T_L{}' = m\times T_L. \quad (24)$$

For dimension chains with multiple dimensions, they should be divided into two chains I and II, trying to obtain an equal sum of the tolerances of their component dimensions using Eq. (25).

chain I chain II

$$\sum_{i=1}^{p}\left|\frac{\partial L}{\partial A_i}\right| T_{A_i} = \sum_{i=p+1}^{m}\left|\frac{\partial L}{\partial A_i}\right| T_{A_i}, \quad (25)$$

where p is the number of component dimensions of one of the two chains obtained from splitting the chain under consideration, and m the number of independent dimensions of the chain under consideration.

In the case of the wheel hub unit under consideration for the axial clearance tolerance equation this looks like the following:

$$\frac{\partial L}{\partial n}T_n + \frac{\partial L}{\partial S}T_S = \frac{\partial L}{\partial m}T_m + \frac{\partial L}{\partial d}T_d + \frac{\partial L}{\partial R}T_R.$$

The equality of parts I and II of the chains could not be achieved, as the following calculation illustrates.

$$1\times0.04+2.375\times0.04 \approx 1\times0.02+3.105\times0.0195+0.73\times0.1,$$

$$0.135 \text{ mm} \neq 0.154 \text{ mm.}$$

Actual tolerance of the closing dimension $T_L{}'$=0.135+0.154= 0.289 mm, and the desired tolerance is T_L=0.020 mm.

In order to meet the condition that the tolerance in a selection group should not be greater than the tolerance of the closing dimension using Eq. (24), the number of selection groups was calculated, which should be no less than 15 [22]:

- executive deviations: $x_2{}'$ = 0.144 mm and $x_1{}'$ = –0.144 mm,
- desired deviations: x_2 = 0.01 mm and x_1 = –0.01 mm.

For the coordinates of the centre of the area of variation the following condition must be met:

$$M_L{}' = M_L, \quad (26)$$

where $M_L{}'$ is coordinate of the centre of the performance tolerance variation area, and M_L coordinate of the centre variation area of the tolerance desired. Using the formulas:

$$M_L = 0.5(x_1+x_2) \text{ and } M_L{}' = \sum_{i=1}^{m}\left(\frac{\partial L}{\partial A_i}\right)M_{A_i}, \quad (27)$$

where x_1, x_2 are desired deviations, and M_{A_i} are coordinates of the centres of the area of variation of the component dimensions before selection M_L=0.015 mm, and $M_L{}'$ = 0 mm.

A zero value was obtained due to symmetrically distributed deviations for each component dimension. In the present case, the condition of equality of the coordinates of the centres of the areas of variation $M_L{}'=M_L$ is not satisfied. The fulfilment of the conditions of equality of the coordinates of the area of variation Eq. (26) and equality of tolerance Eq. (25) gives confidence in the constancy of the resultant dimensions in each selection group [22]. In order to meet this requirement, the dimensional deviations of some component cells were changed and recalculated. New deviation values for the ball diameter d were adopted on the basis of experience and deviation calculations were carried out for R and S. Table 3 summarises the previous and new tolerance values.

$$M_L' = 0.5\sum_{i=1}^{m}\left(\frac{\partial L}{\partial A_i}\right)\left(x_{1i}+x_{2i}\right)$$
$$= 0.50\times\left(-3.105\times\left(-0.005\right)+0.73\times\left(x_{1R}+x_{2R}\right)\right)=0.015 \text{ mm},$$

and $T_R=0.08=x_{2R}-x_{1R}$, we get $x_{1R}=-0.03$ mm and $x_{2R}=0.05$ mm.

Table 3. Previous and new tolerance values

Dimension	Nominal value	Design tolerance	New tolerance
r_z [mm]	6.730	± 0.025	+0.025, –0.015
r_w [mm]	6.540	± 0.025	+0.025, –0.015
$R = r_w+r_z$ [mm]	13.270	± 0.050	+0.050, –0.030
d [mm]	12.700	± 0.0097	+0.001, –0.006
d_z [mm]	65.878	± 0.020	± 0.013
d_w	40.211	± 0.020	± 0.013
$S = 0.5(d_z–d_w)$ [mm]	12.834	± 0.020	± 0.0125

After this change, equal values of the coordinates of the centres of the tolerance variation areas were obtained. Rechecking the condition of equality of tolerance of the two parts of the split dimensional chain.

$1\times0.04+2.375\times0.04\approx1\times0.02+3.105\times0.007+0.73\times0.08,$
$0.135\neq0.1.$

The condition of equality of tolerance is not fulfilled, which means that the axial clearance will not reach the value between 0.05 and 0.025 mm. To avoid this, the tolerance of dimension S was changed without changing the equality of values of the centres of the tolerance variation areas of part I of the chain. For this purpose, the necessary tolerance for S was calculated.

The tolerance of Part I is equal to $1\times0.04+2.375\times T_S' = 0.1$ so $T_S' = 0.025$. Then S will be $12.834^{\pm0.0125}$ mm.

Calculation of actual tolerance for new tolerances on independent dimensions $T_L' = 1\times0.04+1\times0.02+2.375\times0.0125+3.105\times0.007+0.73\times0.08 = 0.17$ mm. For an actual axial clearance tolerance of 0.17 mm, the number of selection groups will be 9.

4 DISCUSSION

Wheel hub units have numerous benefits and as such they are replacing traditional wheel bearings in motor vehicles. To design them, engineers now commonly apply GPS principles since these help them better reflect their intentions and prevent drawing interpretation errors. The mathematical model proposed in this article was used to determine the amount axial clearance and the optimal tolerances of toleranced dimensions for a generation 2 wheel hub unit with a dual-row angular contact ball bearing. In this case the axial clearance determined using the dimensional chain equation was 3.3 µm greater than the value assumed by the designer. The tolerance values obtained with the model were validated by comparing them with the values assumed by the designer for this product. These are significantly smaller than the tolerances adopted by the designer. This indicates that it is necessary to change the tolerances of the component dimensions and manufacture the product under interchangeability conditions using the selective compensation approach. In the case under consideration, the division of the independent dimensions into nine selection groups allows components to be manufactured with tolerances that are achievable, economically justified and assembly guaranteed to meet the designer's requirements.

5 CONCLUSIONS

The proposed solution of using a mathematical dimensional chain model in the design of a second-generation bearing hub is an alternative to the frequently used product and process design method based on designer experience, numerous tests and at considerable cost. Its use will reduce the time and cost associated with designing a bearing hub and putting it into production. The proposed solution will still be documented and validated in series production of bearing hubs which is planned in the near future.

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Data availability *Data supporting the proposed solution will be available from authors K Kuźmicki and M Gajur.*

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Matematični model dimenzijske verige za enoto kolesnega pesta 2. generacije

POVZETEK *Članek predstavlja pregled konstrukcijskih rešitev pesta kolesa ter predlaga matematični model dimenzijske verige in enačbo za izračun tolerance za izračun osne zračnosti za sklop pesta kolesa 2. generacije s krogličnimi ležaji. Analiza dimenzijske verige in njena sinteza sta izvedeni z uporabo treh metod delne zamenljivosti. Predlagana je bila možnost izdelave ležaja pesta z uporabo izbirne kompenzacije. Opravljeni razmisleki predstavljajo alternativo sedanji metodi načrtovanja procesa, ki temelji na številnih poskusih in precejšnjih stroških.*

Ključne besede *kotalni ležaji, dimenzijska veriga, tolerančna formula, osna zračnost, enota kolesnega pesta*